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LETTER TO THE EDITOR

Path integral evaluation of the Bloch density matrix for an oscillator in an electric and magnetic field

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Abstract. The Bloch density matrix for an electron in a three-dimensional oscillator potential in the presence of parallel electric and magnetic fields is presented in cylindrical coordinates.

There has been renewed interest in the evaluation of the Bloch density matrix for an electron in an oscillator well (March and Tosi 1985, Manoyan 1986). A number of years ago it was shown (Glasser 1964) that the path integral representing the Feynman propagator could be evaluated for a quadratic Hamiltonian by representing the path as a Fourier series and integrating over the coefficients. This was carried out explicitly for an electron in a magnetic field in *rectangular coordinates.* A trivial modification of that work (completing the square and translating the coordinates) leads to the propagator for the Hamiltonian

$$\mathscr{H} = (\boldsymbol{p} - (e/c)\boldsymbol{A})^2/2m + \tfrac{1}{2}\kappa r^2 + \boldsymbol{F}\cdot\boldsymbol{r} + \boldsymbol{\mu}_0\boldsymbol{\sigma}\cdot\boldsymbol{H} \qquad \boldsymbol{A} = \tfrac{1}{2}\boldsymbol{r}\times\boldsymbol{H}. \tag{1}$$

The corresponding density matrix (as a 2×2 matrix with respect to spin $\sigma = \pm 1$) is

$$\begin{aligned} C_{\sigma\sigma'}(\boldsymbol{r}\boldsymbol{r}'\beta) = {} & (m\omega_0/2\pi\hbar)^{1/2}(m\Omega/2\pi\hbar)\delta_{\sigma\sigma'}(\operatorname{cosech}\hbar\omega_0\beta)^{1/2}\operatorname{cosech}\hbar\Omega\beta \\ & \times\exp[(F^2\beta/2m\omega_0^2) - \hbar\omega_0\sigma s]\exp\{-(m\omega_0/2\hbar)|[(z+z_0)^2 \\ & +(z'+z_0)^2]|\coth\hbar\omega_0\beta - 2(z+z_0)\operatorname{cosech}\hbar\omega_0\beta\}| \\ & \times\exp(-(m\Omega/2\hbar)\{(\rho^2+\rho'^2)\coth\hbar\Omega\beta \\ & -2\rho\rho'\cosh[\hbar\omega_0\beta - \mathrm{i}(\phi-\phi)]\operatorname{cosech}\hbar\Omega\beta\}) \end{aligned} \tag{2}$$

for $\boldsymbol{F}\|\boldsymbol{H}$, where both fields are along the z axis and we have introduced the quantities

$$\omega_c = eH/2mc \qquad \omega_0^2 = \kappa/m \qquad \Omega^2 = \omega_c^2 + \omega_0^2 \qquad z_0 = F/m\omega_0^2.$$

For simplicity the formula has been expressed in cylindrical coordinates. Other field configurations are discussed by Schulman (1981). Equation (2) reduces to Manoyan's equation (15) for $F\to 0$ and has applications to the study of the polarisability of small metal particles.

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